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Abstract. Cornish argues that the 'interaction theory' of electrodynamics may provide a much simpler description of radiation processes than that of conventional Maxwell–Lorentz electrodynamics, but is unable to derive it from a variational principle. In this paper we exhibit the fact that a variational principle exists which contains the interaction theory as a special case.

1. Introduction

Cornish (1965) has shown how to develop various classes of electromagnetic theories by assuming that Maxwell's equations hold for point charges, and then regarding the equations of motion of the point charges as derivable from various modified forms of the energy momentum tensor.

In particular, he has discussed the 'interaction theory', in which the equations of motion do not contain conventional radiation reaction terms, but account for mutual retardation effects between the various point charges in the system. He has argued that the 'interaction theory' has advantages over that of the conventional Maxwell–Lorentz electrodynamics, and can provide a much simpler description of radiation processes. However, he is unable to derive this theory from a variational principle, which he feels is a chief disadvantage.

The purpose of this paper is to exhibit the fact that a variational principle exists which contains the 'interaction theory' as a particular member of a general class of electrodynamic theories.

2. A Lagrangian formulation for 'interaction theory' electrodynamics

We shall consider a model universe filled with N charged point particles. The action principle which contains the 'interaction theory' of electrodynamics of this system can be written as

$$I = \int \sum_{K=1}^{N} \mathrm{d}\tau_{(K)}\, \tfrac{1}{2} m_{(K)} u_{\mu(K)} u^{\mu}_{(K)} + \int \mathrm{d}x^4 \sum_{K=1}^{N} \sum_{J \neq K}^{N} \left(\tfrac{1}{4} F^{\mu\nu}_{(K)} F_{\mu\nu(J)} + J^{\mu}_{(K)} A_{\mu(J)}\right) \tag{1}$$

where

$$x^{\mu}_{(K)},\ \mathrm{d}\tau_{(K)},\ u^{\mu}_{(K)} \text{ and } m_{(K)}$$

are the Kth particle's trajectory, proper time, 4-velocity and mass, respectively. $F^{\mu\nu}_{(K)}$ is the Kth particle's Maxwell field where

$$F^{\mu\nu}_{(K)} = A^{\mu,\nu}_{(K)} - A^{\nu,\mu}_{(K)} \tag{2}$$

and

$$J^{\mu}_{(K)} = \frac{q(K)}{c} \frac{\mathrm{d}x^{\mu}_{(K)}}{\mathrm{d}t} \delta^3\{x - x_K(t)\} \tag{3}$$

is the Kth particle's 4-current density. Setting $\delta I = 0$ with respect to $\delta x^{\mu}_{(K)}$ and $\delta A^{\mu}_{(K)}$ yields the equations of motion

$$\frac{\mathrm{d}p^{(K)}_{\mu}}{\mathrm{d}t} = \left(\frac{q(K)}{c}\right) \frac{\mathrm{d}x^{\nu}_{(K)}}{\mathrm{d}t} \left(\sum_{J \neq K}^{N} F^{(J)}_{\mu\nu}\right) \tag{4}$$

and the 'Maxwell equations'

$$\sum_{J \neq K}^{N} F_{\mu\nu}^{(J),\nu} = \sum_{J \neq K}^{N} J_{\mu}^{(J)} \tag{5}$$

$$\tilde{F}_{\mu\nu}^{(J),\nu} \equiv 0. \tag{6}$$

If we choose the Lorentz gauge

$$A_{\mu}^{(J),\mu} = 0 \tag{7}$$

then (5) becomes

$$\sum_{J \neq K}^{N} \Box A_{\mu}^{(J)} = \sum_{J \neq K}^{N} J_{\mu}^{(J)}. \tag{8}$$

If there are no less than two charged particles in the system, then $N \geqslant 2$ and (7) can be solved algebraically for the individual $\Box A_{\mu}^{(J)}$ terms (since the determinant of their coefficients is non-zero) as

$$\Box A_{\mu}^{(J)} = J_{\mu}^{(J)}. \tag{9}$$

We can solve (9) as

$$A_{\mu}^{(J)}(x) = \int \mathrm{d}x^4 \sum_{K=1}^{N} D^{JK}(x-x')J_{\mu}^{(K)}(x') \tag{10}$$

by using a Green function $D^{JK}(x-x')$ obeying the equation

$$\Box D^{JK}(x-x') = \delta^4(x-x')\delta^{JK}. \tag{11}$$

In (10) we exclude homogenous solutions, unconnected from currents, since they should not play a role in the 'mutual interactions' which act as building blocks in the 'interaction theory'. The most general solution to (11) is

$$D^{JK}(x-x') = D_{+}(x-x')\delta^{JK} + \lambda^{JK}D_{-}(x-x') \tag{12}$$

where

$$D_{\pm}(x-x') = \tfrac{1}{2}\{D_{\mathrm{ret}}(x-x') \pm D_{\mathrm{adv}}(x-x')\} \tag{13}$$

and λ^{JK} is an arbitrary constant matrix, to be determined. Substituting (10) and (12) into (4) yields the equations of motion

$$\frac{\mathrm{d}p_{\mu}^{(K)}}{\mathrm{d}t} = \left\{\frac{q(K)}{c}\right\}\frac{\mathrm{d}x_{(K)}^{\nu}}{\mathrm{d}t}\left\{\sum_{J \neq K}^{N} (F_{\mu\nu}^{(J)}(+)) + \sum_{J \neq K}^{N} \sum_{L=1}^{N} \lambda^{JL} F_{\mu\nu(-)}^{(L)}\right\}. \tag{14}$$

This equation of motion, with (10) and (2), represents a class of electromagnetic theories (parameterized by λ^{KL}) for which the 'interaction theory', discussed by Cornish, is a member. In particular, if $\lambda^{KL} = \delta^{KL}$ is chosen then (14) becomes the 'interaction theory' equations of motion

$$\frac{\mathrm{d}p_{\mu}^{(K)}}{\mathrm{d}t} = \frac{q(K)}{c}\left(\frac{\mathrm{d}x^{\nu}}{\mathrm{d}t}\right) \sum_{J \neq K}^{N} \{F_{\mu\nu(\mathrm{ret})}^{(J)}\}, \qquad N \geqslant 2. \tag{15}$$

Of course, if $N = 1$ then we must go back to (1), which implies that

$$\frac{\mathrm{d}p_{\mu}^{(1)}}{dt} = 0. \tag{16}$$

This means that a particle alone in the model universe cannot interact with itself. The conservation laws, obtained from (5) and (15), with $\lambda^{KJ} = \delta^{KJ}$, are

$$T_{\mu\nu}{}^{,\nu} = \sum_{K=1}^{N} \sum_{J \neq K}^{N} J_{\nu}^{(K)} F_{\mu}^{(J)\nu} \tag{17a}$$

$$T_{\mu\nu} = \sum_{K=1}^{N} \sum_{J \neq K}^{N} (\tfrac{1}{4} g_{\mu\nu} F_{\alpha\beta}^{(K)} F_{(J)}^{\alpha\beta} + F_{\mu\alpha}^{(K)} F_{(J)}^{\alpha\nu}) \tag{17b}$$

in agreement with those used by Cornish to calculate dipole radiation and collision processes.

3. Conclusions

We have shown that the 'interaction theory' of electrodynamics discussed by Cornish (1965) is a member of a generalized class of theories which are derivable from a single variational principle, thus removing the chief disadvantage of the development of the theory. However, in the process we have revealed an entire class of electrodynamic theories (parameterized by λ^{KJ}) for which the 'interaction theory' is but one member (corresponding to $\lambda^{KJ} = \delta^{KJ}$). A study of the physical implications of this wider class of solutions and its relationship to the process of electromagnetic measurement has been discussed elsewhere by Leiter (1969 a, b). It has been shown to contain a theory similar to that of Wheeler–Feynman electrodynamics (Wheeler and Feynman 1945, 1959), with the distinct advantage that retardation and radiation reaction can be accounted for without the use of any 'complete absorber' condition. In addition, the infinities associated with self-interaction are absent from this theory (as they are for the entire class of parameterized theories).

Now, it is true that this kind of formalism, requiring an external condition restricting the class of allowed solutions, can lead to considerable difficulties if a conventional 'second quantization' is attempted. However, there is no difficulty if one merely attempts to replace the point-mechanical degrees of freedom with wave-mechanical degrees of freedom. Theories of this fashion have been constructed from Wheeler–Feynman electrodynamics by Hoyle and Narlikar (1969) (utilizing a Feynman–Hibbs path integral approach to quantization). Also a wave-mechanical extension of the parameterized theory presented here has been carried out by Leiter (1969 c, d) and has been shown to be able to predict the same results as that of Hoyle and Narlikar (but without the need for a complete absorber condition or steady-state cosmology). Hence it is a possibility that the extension of this class of theories (or a member of this class) into a quantum framework may lead to a reformulation of electrodynamics and quantum theory, which avoids the divergence problems which make the present form of quantum electrodynamics an inconsistent theory.

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